# Mathematics 122

**Quiz #36** Name: Key

***You must show your work to get full credit.***

A tank holding 100 gallons starts with 15 lbs of salt dissolved in it. Pure water starts to flow into the tank at 3 gallons/hour and well mixed water flows out at the same rate. Let $S(t)$ be the amount of salt in the tank $t$ hours after the water starts to flow into the tank.

1 pt (1) What is the concentration of the salt (that is the amount per gallon) at time $t$?

There are $S(t)$ lbs of salt and 100 gallons so it is ↑ Concentration is $\frac{S(t)}{100}$

1 pt (2) What is the rate that the salt is leaving the tank expressed in terms of $S(t)$ and the volume and flow rate?

There are $\frac{S(t)}{100}$ lbs of salt/gallon and water is leaving at 3 gal/hr. So rate is $-3\frac{S(t)}{100}$ lbs/hr. Rate is $-\frac{3}{100}S(t)$

1 pt (3) Write a differential equation for $S(t)$. $\frac{dS}{dt} = -\frac{3}{100}S$

$\frac{dS}{dt}$ = rate of change of salt, but by (2) this is $-\frac{3}{100}S$

1 pt (4) Solve this differential equation. $S(t) = 15e^{-\frac{3}{100}t} = 15e^{-.03t}$

$\frac{dS}{dt} = -\frac{3}{100}S(t)$ $S(0) = 15$ so ↑

1 pt (5) How long before only 2 lbs of the salt are left?

Set $S(t) = 15e^{-.03t} = 2$

$e^{-.03t} = 2/15$

$-.03t = \ln(2/15)$

$t = \frac{\ln(2/15)}{-.03} = 67.16$ hours

$t =$ 67.16 hours.