Homework assigned Wednesday, October 6

What we saw in class today is that if $i = \sqrt{-1}$, then

$$e^{a+bi} = e^a e^{bi} = e^a \left(\cos(b) + i\sin(b)\right)$$

and then if r a complex number, then

$$\frac{d}{dx} e^{rx} = r e^{rx}.$$

We used this to show that if

$$ay'' + by' + cy = 0 \tag{1}$$

has characteristic equation

$$ar^2 + br + c = 0$$

that has complex roots

$$r_1 = \alpha + i\beta, \qquad r_2 = \alpha - i\beta$$

then the general solution to (1) is

$$y_{\text{gen}} = c_1 e^{\alpha x}\cos(\beta x) + c_2 e^{\beta x}\sin(\beta x).$$

As an example consider

$$y'' - 4y + 13y = 0$$

the characteristic equation is

$$2r^2 + 2r + 1 = 0$$

which has solutions

$$r = 2 + 3i, \qquad r = 2 - 3i$$

so the general solution is

$$y_{\text{gen}} = c_1 e^{2x}\cos(3x) + c_2 e^{2x}\sin(3x).$$

Using this you should be able to do

- Page 183, 8, 9, 22, 23.
- Solve the following intial value problems:
  (a) $y'' - 2y' + 5y = 0$, $y(0) = 1$, $y'(0) = -1$.
  (b) $y'' + 2y' + 10y = 0$, $y(0) = 3$, $y'(0) = 2$.
  (c) $y'' - 4y' + 29y = 0$, $y(0) = 6$, $y'(0) = -3$.
  (d) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = -4$.