Mathematics 122

Quiz #26 Name: Key

You must show your work to get full credit.

1. Find the maximum of the function $f(x) = x^3(a-x)$ on $0 \le x \le a$.

$f(x) = x^3(a-x) = ax^3 - x^4$

$f'(x) = 3ax^2 - 4x^3)$

$= x^2(3a-4x)$

At a max. or min. $f'(x) = 0$

so $x^2(3a-4x) = 0$

Thus $x^2 = 0$ (i.e. $x=0$)

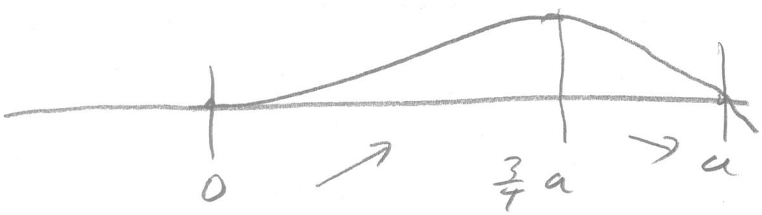
or $3a - 4x = 0$ (i.e. $x = \frac{3}{4}a$)

so $x = \frac{3}{4}a$ is maximizer

maximum is

$f(\frac{3}{4}a) = (\frac{3a}{4})^3 (a - \frac{3}{4}a)$

$= \frac{27a^3}{64} \cdot \frac{a}{4} = \frac{27a^4}{256}$



maximizer is $\frac{3}{4}a$

maximum is

2. Compute the following derivatives.

$f(x) = 2x^2 e^{x^3}$ $f'(x) =$ $(4x+6x^4)e^{x^3}$

$f'(x) = 4xe^{x^3} + 2x^2 e^{x^3}(3x^2)$

$= (4x + 2x^2(3x^2))e^{x^3} = (4x+6x^4)e^{x^3}$

$q = \dfrac{e^u}{u+1}$ $\dfrac{dq}{du} =$ $\dfrac{ue^u}{(u+1)^2}$

$$\frac{dq}{du} = \frac{(e^u)'(u+1) - e^u(u+1)'}{(u+1)^2} = \frac{e^u(u+1) - e^u(1)}{(u+1)^2}$$

$$= \frac{e^u(u+1-1)}{(u+1)^2} = \frac{ue^u}{(u+1)^2}$$