

Quiz #33

Name: key*You must show your work to get full credit.*

1. Let $f(x) = \frac{x^2 3^x}{2 + (1.5)^{x-1}}$. Find the average value of $f(x)$ on the interval $1 \leq x \leq 4$.

The average is _____

$$Av. = \frac{1}{(4-1)} \int_1^4 f(x) dx = \frac{1}{3} \int_1^4 \frac{x^2 3^x}{2 + (1.5)^{x-1}} dx$$

2. Some review of topics that have been troublesome.

(a) Find the derivative of $f(x) = xe^{-ax}$ where a is a constant.

$$\begin{aligned} f'(x) &= 1e^{-ax} + xe^{-ax}(-a) \\ &= e^{-ax}(1 - ax) \end{aligned}$$

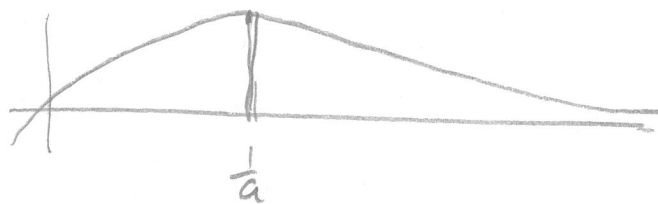
$$f'(x) = \underline{(1 - ax)e^{-ax}}$$

(b) For $f(x) = xe^{-ax}$ find the maximizer and maximum for $x \geq 0$ and where $a > 0$ is constant.

$$\text{Set } f'(x) = (1 - ax)e^{-ax} = 0$$

Maximizer is $\frac{1}{a}$

$$\begin{aligned} \text{Then } 1 - ax &= 0 \\ -ax &= -1 \\ x &= \frac{1}{a} \end{aligned}$$

Maximum is $\frac{1}{ae}$ 

$$\begin{aligned} f'(x) &> 0 \text{ for } x < \frac{1}{a} \\ f'(x) &< 0 \text{ for } x > \frac{1}{a} \end{aligned}$$

$$\begin{aligned} \text{maximum} &= f\left(\frac{1}{a}\right) \\ &= \frac{1}{a} e^{-a\left(\frac{1}{a}\right)} \\ &= \frac{1}{a} e^{-1} = \frac{1}{ae} \end{aligned}$$