Math 554 Homework

Problem 1. Since we spent some time looking at binomial coefficients, here is an interesting formula relating them to derivatives. You know the product rule

$$(fg)' = f'g + fg'$$

Using this it is not hard to check that for the second and thrid derivatives we have

$$(fg)'' = f''g + 2f'g' + fg'', \qquad (fg)''' = f'''g + 3f''g' + 3f'g'' + fg'''.$$

This leads to the conjecture

$$(fg)^{(n)} = \sum_{m=0}^{n} \binom{n}{m} f^{(m)} g^{(n-m)}$$

where $f^{(m)}$ is the m-th derivative of f and by convention $f^{(0)} = f$. Prove this by induction. *Hint:* The only property of the biomomial coefficents you should need is the Pascal property $\binom{n+1}{m} = \binom{n}{m} + \binom{n}{m-1}$.

Problem 2. We will be working with sets and doing operations like taking unions, intersections, and compliments of sets. I am assuming that this is mostly review. In the text look at the section **Some Set Theory** on pages 19 – 21. Do problems 1 and 2 on page 27 of the text to be handed on Monday.

Problem 3. The definitions I want you to know by Monday for the quiz from Definitions 1.3.2 (Page 21) and 1.3.4 (Page 23). In partucular know the definitions of the following

(a) An ε ***neighborhood*** of x_0.
(b) x_0 is an ***interior point*** of S.
(c) The ***interior*** of S.
(d) S is an ***open*** set.
(e) S is a ***closed*** set.
(f) x_0 is a ***limit point*** of S.
(g) x_0 is a ***boundary point*** of S.
(h) The ***boundary***, ∂S, of S.
(i) The ***closure***, $\overline{S}$, of S.