Math 554 Homework

Here are some more ways to construct new continuous functions from old ones.

Proposition 1. *Let f be continuous on the open interval (a, b). Then $|f|$ is continuous on (a, b).*

Problem 1. Prove this. *Hint:* We are given that for each $x_0 \in (a, b)$ that $\lim_{x\to x_0} f(x) = f(x_0)$. We wish to show for $x_0 \in (a, b)$ that $\lim_{x\to x_0} |f(x)| = |f(x_0)|$. So let $\varepsilon > 0$ and let $\delta > 0$ be so that $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$. But note $\big||f(x)| - |f(x_0|\big| \leq |f(x) - f(x_0)|$.

Proposition 2. *For any real numbers a, b*

$$\max\{a, b\} = \frac{a + b + |a - b|}{2} \qquad \textit{and} \qquad \min\{a, b\} = \frac{a + b - |a - b|}{2}.$$

Problem 2. Prove that $\max\{a, b\} = \dfrac{a + b + |a - b|}{2}$. (The proof of formula for min is almost identical, so we will only do one of them).

Proposition 3. *Let f, g be continuous on (a, b). Then the functions p, q defined on (a, b) by*

$$p(x) = \max\{f(x), g(x)\} \qquad \textit{and} \qquad q(x) = \min\{f(x), g(x)\}$$

are also continuous on (a, b).

Problem 3. Prove that p is continuous. (The proof for q is almost identical.) *Hint:* Propositions 1 and 2.

And to pratice induction

Proposition 4. *If $f_1, \ldots, f_n$ are continuous on (a, b) then so is the function g defined by*

$$g(x) = \max\{f_1(x), f_2(x), \ldots, f_n(x)\}.$$

Problem 4. Prove this.

Problem 5. Problem 6 on page 70 of the text.

Problem 6 (Extra Credit). Problem 7 on page 70 of the text.