Mathematics 172 Homework

The last topic we are covering is island biogeography. Let S be the number of species on an island. As a function of S we assume the ***extinction rate***, μ_S, that the number of species that go extinct on the island per unit time is

$$\mu_S = \left(\frac{E}{P}\right) S$$

where E and P are constants that depend on the island. We assume the ***immigration rate***, λ_S, the rate that new species immigrate to the island, is

$$\lambda_S = I - \left(\frac{I}{P}\right) S$$

where I is a constant (the immigration rate of the island when empty).

Then S satisfies the rate equation

$$\frac{dS}{dt} = \lambda_S - \mu_S.$$

1. Show this leads to the rate equation

$$\frac{dS}{dt} = I - \left(\frac{I}{P}\right) S - \left(\frac{E}{P}\right) S.$$

and that the only equilibrium point of this equation is

$$\widehat{S} = \frac{IP}{I+E}$$

and that this will be when $\mu_S = \lambda_S$. □

While this model predicts that the number of species on the island will approach the constant, it does not predict that the same species will stay on the island. The ***turnover rate*** is the number of species that are arriving (or departing) species per unit time when the number of species is at equilibrium.

2. If $\widehat{T}$ is this turnover rate, then $\widehat{T}$ will be the extinction rate, μ_S, when $S = \widehat{S}$. Use this to show that

$$\widehat{T} = \left(\frac{E}{P}\right) \widehat{S} = \frac{IE}{I+E}.$$