Mathematics 172

Quiz # 14 Name: Key

You must show your work to get full credit.

For the initial value problem

$$\frac{dP}{dt} = f(P) \qquad \text{and} \qquad P(0) = P_0$$

Euler's method with step size Δt is

$$\begin{aligned} t_0 &= 0 \\ P_0 &= P_0 \\ t_{k+1} &= t_k + \Delta t \\ P_{k+1} &= P_k + f(P_k)\Delta t. \end{aligned}$$

with the result

$$P(t_k) \approx P_k$$

Consider the initial value problem

$$\frac{dP}{dt} = .05P(10 - P) \qquad \text{and} \qquad P(0) = 8.$$

1. Use Euler's method with three steps of size $\Delta t = .5$ to estimate $P(1.5)$.

$t_1 = 0 + .5 = .5$

$P_1 = 8 + .05(8)(10-8)(.5)$
$= 8.4$

$t_2 = .5 + .5 = 1.0$

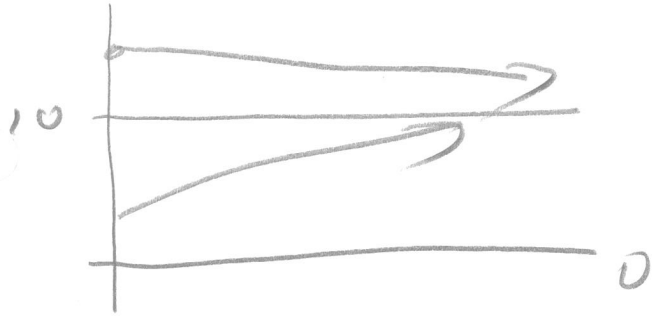
$P_2 = 8.4 + .05(8.4)(10-8.4)(.5)$
$= 8.736$

$P(1.5) \approx$ 9.012057

$t_3 = 1.0 + .5 = 1.5$

$P_3 = 8.736 + .05(8.736)(10-8.736)(.5)$
$= 9.012\,0576$

2. For the same initial value problem estimate $P(100)$.



$P(100) \approx$ 10