Mathematics 172

Quiz # 16 Name: Key

You must show your work to get full credit.

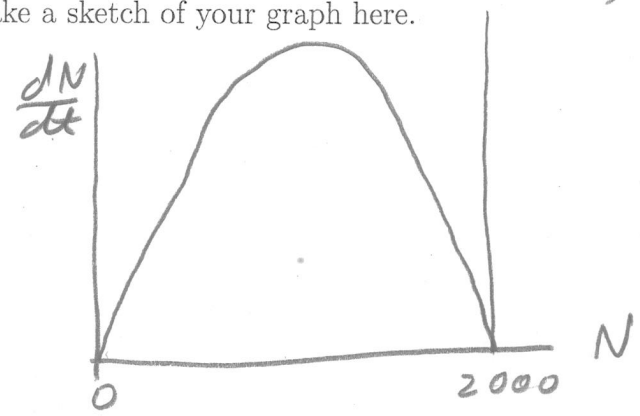
A colony of Daphnia (water fleas) is living in a bucket in the backyard of an aquarist. The intrinsic growth rate of the colony is .4 (Daphnia/day)/Daphnia. The carrying capacity of the bucket is 2,000 Daphnia.

1. Let $N = N(t)$ be the number of Daphnia in the buck after t days. Assuming logistic growth for the colony, write down the rate equation for N.

$$\frac{dN}{dt} = .4N\left(1 - \frac{N}{2000}\right)$$

2. Plot $\frac{dN}{dt}$ as a function of N and make a sketch of your graph here.

$Y_1 = .4X(1 - X/2000)$
$X_{min} = 0$
$X_{max} = 2000$



A some point the aquarist starts starts harvesting the Daphnia to feed to his baby fish.

3. What is the maximum daily rate that the aquarist can harvest the Daphnia without killing off the colony?

The maximum of $\frac{dN}{dt}$ occurs when $N = 1000$ and is $\frac{dN}{dt} = 200$

Maximum harvest rate is: 200

4. If the aquarist harvests at a rate of 100 Daphnia/day what is the new stable equilibrium size of the colony?

The new rate equation is $\frac{dN}{dt} = .4N\left(1 - \frac{N}{2000}\right) - 100$

Equilibrium size is 1707.