Mathematics 172

Quiz # 23 Name: Key

You must show your work to get full credit.

For the Leslie matrix from the last quiz:

$$L = \begin{bmatrix} 0.0 & 6.2 & 39.0 \\ 0.035 & 0.0 & 0.0 \\ 0.0 & 0.62 & 0.0 \end{bmatrix}$$

let use find the per capita growth rate. Assume that

$$\vec{n}(0) = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

is stable age distribution. That is

$$\vec{n}(1) = \lambda\vec{n}(0) \qquad \text{that is} \qquad \vec{n}(1) = \begin{bmatrix} \lambda n_1 \\ \lambda n_2 \\ \lambda n_3 \end{bmatrix}$$

Draw the loop diagram and use it to get anther formula for $\vec{n}(1)$

$$\vec{n}(1) = \begin{bmatrix} 6.2n_2 + 39n_3 \\ .035n_1 \\ .62n_2 \end{bmatrix}$$

Comparing these two formulas for $\vec{n}(1)$ gives three equations. What are they?

$n_2 = \frac{.035}{\lambda} n_1$

$n_3 = \frac{.62}{\lambda} n_2 = \frac{(.62)(.035)}{\lambda^2} n_1$

$\lambda n_1 = \frac{(6.2)(.035)}{\lambda} n_1 + \frac{39(.62)(.035)}{\lambda^2} n_1$

Equation 1 $\lambda n_1 = 6.2n_2 + 39n_3$

Equation 2 $\lambda n_2 = .035 n_1$

Equation 3 $\lambda n_3 = .62 n_2$

Now eliminate n_1, n_2, n_3 from these equation to get an equation for λ. (This is the Euler-Lokta equation.)

divide by λ

Equation for λ $1 = \frac{.217}{\lambda^2} + \frac{.8463}{\lambda^3}$

Solve this using your calculator to get λ and the discrete growth rate $r = \lambda - 1$

$\lambda =$ 1.0222 $\qquad r =$ .0222

Finally give the stable age distribution:

If $n_1 = 1$

$n_2 = \frac{.035}{\lambda} = .03424$

$n_3 = \frac{(.62)(.035)}{\lambda^2} = .02077$

Percent in stage 1 94.2%

Percent in stage 2 3.1%

Percent in stage 3 2.0%