Mathematics 172

Quiz # 9 Name: Key

You must show your work to get full credit.

Let P satisfy the differential equation

$$\frac{dP}{dt} = -.2P + 400$$

1. Find the equilibrium solution.

Equilibrium solution is 2000

Solve $\frac{dP}{dt} = -.2P + 400 = 0$

$-.2P = -400$

$P = \frac{400}{.2} = 2000$

2. Find the solution with $P(0) = 2{,}200$.

$P(0) =$ $2000 + 200e^{-.2t}$

$\frac{dP}{dt} = -.2\left(P + \frac{400}{-.2}\right)$

$\frac{dP}{dt} = -.2(P - 2000)$

Let $y = P - 2000$

$y' = P'$

So $y' = -.2y$

Thus $y = y_0 e^{-.2t}$

Therefore $P = 2000 + y = 2000 + y_0 e^{-.2t}$

$P(0) = 2{,}200 = 2000 + y_0$

$y_0 = 2{,}200 - 2{,}000 = 200$

$P = 2000 + 200e^{-.2t}$