Mathematics 174

Quiz # 10 Name: Key

You must show your work to get full credit.

1. Show that $\sim (p \to q)$ is logically equivalent to $p \wedge \sim q$ by using truth tables.

p	q	$p \to q$	$\sim(p \to q)$	$\sim q$	
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

They are the same

2. (a) State DeMorgan's laws.

$$\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$$
$$\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$$

(b) State the definition of $p \to q$.

$$p \to q \equiv \sim p \vee q$$

(c) Show that $\sim (p \to q)$ is logically equivalent to $p \wedge \sim q$ by using DeMorgan's law.

$$\begin{aligned} \sim (p \to q) &\equiv \sim(\sim p \vee q) \\ &\equiv (\sim\sim p) \wedge (\sim q) \\ &\equiv p \wedge (\sim q) \end{aligned}$$

3. Give the negation of the statement "Some Math174 student will fail the exam."

"All Math174 students will pass the exam"
or
"No Math174 student will fail the exam"

4. Show that if n is a positive integer that $n^2 + 3n + 2$ is a composite number.

$n^2 + 3n + 2 = (n+1)(n+2)$

and as n is positive $n+1 \geq 2$ and $n+2 \geq 3$

Thus n^2+3n+2 factors and therefore is composite

5. Show that if n is even, then $n^2 + 3n + 7$ is odd.

If n is even, then $n = 2k$ for some integer k.

Thus

$$\begin{aligned} n^2+3n+7 &= (2k)^2 + 3(2k) + 7 \\ &= 4k^2 + 6k + 7 \\ &= 4k^2 + 6k + 6 + 1 \\ &= 2(2k^2+3k+3) + 1 = 2(\text{integer}) + 1 \end{aligned}$$

Thus it is odd

6. Show that if d divides n, then d^2 divides $2n^2$.

If $d \mid n$, then $n = qd$ for some integer q.

Then $n^2 = (qd)^2 = q^2 d^2 = (\text{integer}) d^2$

Thus $d^2 \mid n^2$

7. Find $18 \operatorname{div} 5$ and $18 \bmod 5$. Find $-22 \operatorname{div} 5$ and $-22 \bmod 5$.

$18 = 3(5) + 3$

so

$18 \operatorname{div} 5 = 3$

$18 \bmod 5 = 3$

$-22 = -5(5) + 3$

so

$-22 \operatorname{div} 5 = -5$

$-22 \bmod 5 = 3$

8. Write $55.123123123\ldots$ as a ratio of integers. Let $a = 55.123123123$

Then

$$\begin{aligned} 1000a &= 55123.123123123\ldots \\ -\quad a &= 55.123123123\ldots \\ \hline 999a &= 55123 - 55 = 55068 \end{aligned}$$

Thus $a = \dfrac{55068}{999}$