Mathematics 174

Quiz # 11 Name: Key

You must show your work to get full credit.

1. Show that $\sqrt{2}$ is irrational.

Towards a contradiction assume $\sqrt{2}$ is rational. Then $\sqrt{2} = \frac{p}{q}$ with p, q integers and we assume this is in lowest term.

Then $p = \sqrt{2}q$

so $p^2 = 2q^2$

Thus $2 | p^2 \Rightarrow 2 | p$. Thus $p = 2p'$ for some integer p'. then

$$p^2 = (2p')^2 = 4(p')^2 = 2q^2$$

$$\Rightarrow 2(p')^2 = q^2$$

$$\Rightarrow 2 | q^2 \Rightarrow 2 | q$$

Thus $q = 2q'$ for some integer q'

But then $\frac{p}{q} = \frac{2p'}{2q'}$ is not in lowes terms, a contradiction

2. Show that there are infinitely many primes.

Assume towards a contradiction, ~~that~~ There only are finitely many primes $p_1, p_2, \dots, p_n$.

Let $N = p_1 p_2 \cdots p_n + 1$. Then no p_j divides N as $N \bmod p_j = 1$. But N has at least one prime factor, call it p.

Then $p \neq p_j$ for any j as $N \bmod p = 0$.

Thus p is a prime not in our list contradicting that $p_1, \dots, p_n$ is all primes