Mathematics 174

Quiz # 15 Name: Key

You must show your work to get full credit.

There is an island where the only coins have values 3 ¢ and 7 ¢. We will show that it is possible to put exactly n ¢ in a jar for $n \geq 15$.

(a) ***Base case:*** How do we put exactly 14 ¢ in a jar? Put in 2 7¢ coins

(b) ***Induction step:*** If we have a jar with k ¢ and $k \geq 14$ in it, explain how, by removing and adding coins, we can put $(k+1)$ ¢ in the jar.

Case 1 There are 2 or more 3¢ coins in the jar. Then remove two of them and replace with a 7¢ coin. We then have $k - 2(3) + (7) = k+1$ ¢.

Case 2 There are only 1 or no 3¢ coins in the jar. Then the rest is made up of 7¢ coins with accounts of at least $k - 3 \geq 14 - 3 = 11$ ¢. So there has to be at least 2 7¢ coins. Remove them and add in 5 3¢ coins. This gives

$k - 2(7) + 5(3) = k - 14 + 15 = (k+1)$ ¢.