Mathematics 174

Quiz # 16 Name: Key

You must show your work to get full credit.

1. A store has gift certificates for \$4 and \$5. Here we show that it possible to give a gift of exactly \$$n$ for $n \geq 15$.

(a) *Base case:* Explain how to give gift of exactly \$15. 3 \$5 certificates

(b) Now assume that we have a gift of exactly \$$k$ with $k \geq 15$.

Case 1: In the collection of gift certificates making up our gift of k there is at least one \$4 gift certificate. Explain how to get a gift of exactly \$$(k+1)$. Remove a \$4 certificate and replace it with a \$5 certificate

Case 2: In the collection of gift certificates making up our gift of \$$k$ there are no \$4 gift certificates and therefore all the gift certificates in our gift are 5 gift certificates. As $k \geq 15$ this means there are at least 3 \$5 gift certificates. Now explain how to get a gift of exactly \$$(k+1)$.

Remove 3 \$5 certificates and replace them with 4 \$4 certificates

2. Use induction to show $1 + 5 + 5^2 + \cdots + 5^n = \frac{5^{n+1}-1}{4}$. (If you prefer summation notation this can also be written as $\sum_{j=0}^{n} 5^j = \frac{5^{n+1}-1}{4}$.)

Base case $n=0$ $5^0 = 1 = \frac{5^{0+1}-1}{4}$

Induction step: Assume $1+5+ \cdots + 5^k = \frac{5^{k+1}-1}{4}$ (*)

(WTS $1+5+\cdots+5^k+5^{k+1} = \frac{5^{k+2}-1}{4}$)

Add 5^{k+1} to both sides of (*)

$$1+5+\cdots+5^k+5^{k+1} = \frac{5^{k+1}-1}{4} + 5^{k+1}$$
$$= \frac{5^{k+1}-1}{4} + \frac{4\cdot 5^{k+1}}{4}$$
$$= \frac{5^{k+1}-1+4\cdot 5^{k+1}}{4}$$
$$= \frac{(1+4)5^{k+1}-1}{4}$$
$$= \frac{5\cdot 5^{k+1}-1}{4} = \frac{5^{k+2}-1}{4}$$