Mathematics 174

Quiz # 17 Name: key

You must show your work to get full credit.

1. Show that $11^n - 1$ is divisible by 10 for all $n \geq 0$.

Base case $11^0 - 1 = 1 - 1 = 0$ is divisible by 10

Induction step show $10 \mid (11^k - 1) \Rightarrow 10 \mid (11^{k+1} - 1)$

Assume $10 \mid (11^k - 1)$. Then $11^k - 1 = 10\ell$ for some ℓ. That is $11^k = 10\ell + 1$

$$\begin{aligned} \text{Then } 11^{k+1} - 1 &= 11 \cdot 11^k - 1 \\ &= 11 \cdot (10\ell + 1) - 1 \\ &= 110\ell + 11 - 1 \\ &= 110\ell + 10 \\ &= 10(11\ell + 1) \end{aligned}$$

Thus $10 \mid (11^{k+1} - 1)$

2. Show that $n^3 < 3^n$ for all $n \geq 4$.

Base case $n = 4$ $4^3 = 64 < 3^4 = 81$

Induction step Show $k^3 < 3^k \Rightarrow (k+1)^3 < 3^{k+1}$

Assume $k^3 < 3^k$. Multiple both sides of this by $(1 + \frac{1}{k})^3 = (\frac{k+1}{k})^3$

$$k^3 \left(\frac{k+1}{k}\right)^3 < 3^k \left(\frac{k+1}{k}\right)^3$$

ie $(k+1)^3 < 3^k (1 + \frac{1}{k})^3$ (*)

As $k \geq 4$, $\frac{1}{k} \leq \frac{1}{4}$ so $(1 + \frac{1}{k})^3 \leq (1 + \frac{1}{4})^3 = (\frac{5}{4})^3 = \frac{125}{64} < 3$,

use this in (*) to get

$$(k+1)^3 < 3^k \cdot 3 = 3^{k+1}.$$