Mathematics 174

Quiz # 20 Name: Key

You must show your work to get full credit.

1. Assume that $\sqrt{5}$ is irrational. Show that $3 + \frac{1}{2}\sqrt{5}$ is irrational.

Towards a contradiction assume $3+\frac{1}{2}\sqrt{5}$ is rational,

then $3+\frac{1}{2}\sqrt{5} = \frac{a}{b}$ with a,b integers.

solve for $\sqrt{5}$: $\frac{1}{2}\sqrt{5} = \frac{a}{b} - 3 = \frac{a-3b}{b}$

$\sqrt{5} = \frac{2a-6b}{b} = \frac{\text{integer}}{\text{integer}}$

which is impossible as $\sqrt{5}$ is irrational.

2. Show that if 5 divides n then 5 does not divide $n+3$.

Towards a contradiction assume $5 \mid (n+3)$.

Then $(n+3) = 5k$ for some k. But $5 \mid n$

so $n = 5l$ for some l.

Thus $3 = (n+3) - n = 5k - 5l = 5(k-l)$.

This implies 5 divides 3 which is a contradiction.

3. For the following sequence $b_0, b_1, b_2, \ldots$ give a formula for b_n $1, -3, 5, -7, 9, -11, 13, -15, 17, \ldots$

$b_n = \underline{(-1)^n}(2n+1)$

4. Compute the following:

$$\sum_{k=-1}^{3} k^3 = \underline{\quad 35 \quad}$$

$$\prod_{k=2}^{9} \frac{k}{k-1} = \underline{\quad 9 \quad}$$

$$\sum_{k=1}^{10} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right) = \underline{\quad \frac{120}{121} \quad}$$

$$\sum_{k=-1}^{3} k^3 = (-1)^3 + 0^3 + 1^3 + 2^3 + 3^3 = -1 + 0 + 1 + 8 + 27 = 35$$

$$\prod_{k=2}^{9} \frac{k}{k-1} = \frac{\cancel{2}}{1}\frac{\cancel{3}}{\cancel{2}}\frac{\cancel{4}}{\cancel{3}}\frac{\cancel{5}}{\cancel{4}}\frac{\cancel{6}}{\cancel{5}}\frac{\cancel{7}}{\cancel{6}}\frac{\cancel{8}}{\cancel{7}}\frac{9}{\cancel{8}} = 9$$

$$\sum_{k=1}^{10}\left(\frac{1}{k^2} - \frac{1}{(k+1)^2}\right) = \left(\frac{1}{1^2} - \cancel{\frac{1}{2^2}}\right) + \left(\cancel{\frac{1}{2^2}} - \cancel{\frac{1}{3^2}}\right) + \cdots + \left(\cancel{\frac{1}{9^2}} - \cancel{\frac{1}{10^2}}\right) + \left(\cancel{\frac{1}{10^2}} - \frac{1}{11^2}\right)$$

$$= 1 - \frac{1}{11^2} = 1 - \frac{1}{121} = \frac{120}{121}$$