5. Show that using only 3¢ and 4¢ stamps it is possible to put exactly n¢ on a letter for any $n \geq 8$.

Base case: $8¢ = 2 \times 4¢$.

Assume we can put exactly k¢ on a letter, $k \geq 8$. If one of the stamps is a 3¢, then take it off and add a 4¢ to get $(k+1)$¢.

If no 3¢ were used, then all the stamps are 4¢. As $k \geq 8$ there are at least two of them. So remove two 4¢ and add 3×3¢ to get $(k+1)$¢. This closes the induction.

6. Use induction to show that $\sum_{j=0}^{n} 2^j = 2^{n+1} - 1$.

Base case $n=1$ $\sum_{j=0}^{1} 2^j = 2^0 + 2^1 = 1 + 2 = 3 = 2^{1+1} - 1$

Induction step Assume $\sum_{j=1}^{k} 2^j = 2^{k+1} - 1$

Add 2^{k+1} to both sides

$$\sum_{j=0}^{k} 2^j + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$$

$$\sum_{j=0}^{k+1} 2^j = 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1$$

which completes the induction.

7. Use induction to show that $4^n - 1$ is divisible by 3 for all $n \geq 1$.

Base case $n = 1$. $4^1 - 1 = 3$ is divisable by 3.

Assume $4^k - 1$ is divisable by 3. Then for some integer l $4^k - 1 = 3l$. Therefore $4^k = 3l + 1$.

Thus
$$\begin{aligned} 4^{k+1} - 1 &= 4 \cdot 4^k - 1 = 4(3l+1) - 1 \\ &= 12l + 4 - 1 \\ &= 12l + 3 \\ &= 3(4l+1) = 3\,(\text{Integer}) \end{aligned}$$

Thus $4^{k+1} - 1$ is divisable by 3. This completes the induction.