8. Let $c_1, c_2, c_2, \ldots$ be defined by

$$c_k = 2c_{k-1} + k \qquad \text{for} \qquad k \geq 2, \text{ and } c_1 = 1$$

Write the first fives terms in the sequence.

$c_1 =$ 1 $\quad c_2 =$ 4 $\quad c_3 =$ 11 $\quad c_4 =$ 26 $\quad c_5 =$ 57

$c_1 = 1$

$c_2 = 2 \cdot 1 + 2 = 4$

$c_3 = 2 \cdot 4 + 3 = 11$

$c_4 = 2 \cdot 11 + 4 = 22 + 4 = 26$

$c_5 = 2 \cdot 26 + 5 = 52 + 5 = 57$

9. Show that $a_n = c2^n$ is a solution to

$$a_n = 5a_{n-1} - 6a_{n-2}$$

for $n \geq 0$ and where c is a constant.

We plug $a_n = c\,2^n$ into $5a_{n-1} - 6a_{n-2}$ and show it reduces to a_n

$$\begin{aligned}
5a_{n-1} - 6a_{n-2} &= 5c\,2^{n-1} - 6c\,2^{n-2} \\
&= 5c\,2\,2^{n-2} - 6c\,2^{n-2} \\
&= (5c \cdot 2 - 6c)\,2^{n-2} \\
&= (10c - 6c)\,2^{n-2} \\
&= (4c)\,2^{n-2} \\
&= 2^2 c\,2^{n-2} \\
&= c\,2^n = \underline{\underline{a_n}}
\end{aligned}$$