Mathematics 174

You must show your work to get full credit.

1. List all the 2-combinations of $\{a, b, c, d\}$ and use this to find the value of $\binom{4}{2}$ .vfill

$\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}$ $\binom{4}{2} = $ ___6___

2. What is the equation relating $P(20, 7)$ and $\binom{20}{7}$.

$$P(20,7) = 7! \binom{20}{7} \quad \text{or} \quad \binom{20}{7} = \frac{1}{7!} P(20,7)$$

3. Find the following:

$\binom{5}{0} = $ ___1___ $\binom{5}{1} = $ ___5___ $\binom{5}{2} = $ ___10___ $\binom{5}{3} = $ ___10___ $\binom{5}{4} = $ ___5___ $\binom{5}{5} = $ ___1___

4. A student council has 20 students.

(a) How many committees of size 8 can be formed?

$$\binom{20}{8} = 125,970$$

(b) Two members are not allowed two be on the same committee. How many committees of size 8 can be form with this restrictions?

$$(\# \text{ with } A \text{ not } B) + (\# B \text{ not } A) + \#(\text{neither})$$
$$= \binom{18}{7} + \binom{18}{7} + \binom{18}{8}$$

$$\binom{18}{7} + \binom{18}{7} + \binom{18}{8} = 107,406.$$

(c) Two of the council members insist on being ~~committee~~ together. So they are either both on the committee or both off the committee. How many committees of size 8 can be made with this restriction?

$$(\# \text{ both on}) + (\# \text{ both off})$$
$$= \binom{18}{6} + \binom{18}{8} =$$

$$\binom{18}{6} + \binom{18}{8} = 62,322.$$

(d) If there are 9 men and 11 women on the council, then how many (i) How many committees of size 8 have 3 men and 5 women? (ii) How many committees of size 8 have at least one woman?

(i) $\binom{9}{3}\binom{11}{5} = 38,808.$

(ii) $\binom{20}{8} - \binom{9}{8} = 125,962.$
Total # − all men

(e) If the council has 4 Freshman, 6 sophomores, 3 juniors, and 7 seniors, then how many committees can be made up with two members from each class.

$$\binom{4}{2}\binom{6}{2}\binom{3}{2}\binom{7}{2} = 5,670.$$