Mathematics 174

Quiz # 7 Name: Key

[You must show your work to get full credit.]

1. Write $a = 34.3737373737\ldots$ as a ratio of two integers.

The ratio is. $\frac{3403}{99}$

$100a = 3437.37373737\ldots$

$-\quad a = 34.37373737\ldots$

$99a = 3403.0000\ldots$

$\text{So } a = \frac{3403}{99}$

2. Show if r is a rational number and $r \neq 1$, then $\frac{r}{1-r}$ is rational.

If r is a rational number, then

$r = \frac{a}{b}$ with a, b integers and $b \neq 0$.

As $r \neq 1$ we also have $a \neq b$.

$$\frac{r}{1-r} = \frac{\frac{a}{b}}{1-\left(\frac{a}{b}\right)} = \frac{\left(\frac{a}{b}\right)b}{\left(1-\left(\frac{a}{b}\right)\right)b}$$

$$= \frac{a}{b-a} = \frac{\text{integer}}{\text{integer}}$$

Also $b - a \neq 0$ as $a \neq b$. Thus

$\frac{r}{1-r}$ is a ratio of integers and thus an integer.