Mathematics 300

Quiz 10 **Name:** **Answer Key**

You must show your work to get full credit.

1. (a) Show that $\tilde{a} = 3$ is an inverse mod 11 of $a = 4$.

Solution. Recall that for $\tilde{a}$ to be the inverse of $a \bmod n$ just means that $\tilde{a} \cdot a \equiv 1 \mod n$. In our case we have

$$\begin{aligned} 4 \cdot 3 &\equiv 12 \mod 11 \\ &\equiv 1 \mod 11 \end{aligned}$$

and therefore $\tilde{4} = 3 \mod 11$. □

(b) Use your result from part (a) to prove: If $3x \equiv 5 \mod 11$, then $x \equiv 9 \mod 11$.

Solution. Multiply both sides of $3x \equiv 5 \mod 11$ by 4 to get

$$4 \cdot 3x \equiv 4 \cdot 5 \mod 11$$

that is

$$12x \equiv 20 \mod 11.$$

But $12 \equiv 1 \mod 11$ and $20 \equiv 9 \mod 11$ so we end up with

$$x \equiv 9 \mod 11.$$

as required. □

2. Let a and b be integers not both zero and let $d = \gcd(a, b)$. Show that for any integers x and y that $d \mid (ax + by)$.

Solution. As $d = \gcd(a, b)$ we have that d is a divisor of both a and b. That is $d \mid a$ and $d \mid b$. Therefore there are integers k and ℓ such that

$$a = kd \qquad \text{and} \qquad b = \ell d.$$

Therefore

$$ax + by = (kd)x + (\ell d)y = (kx + \ell y)d = md$$

where m is the integer $m = kx + \ell y$. Therefore $d \mid (ax + by)$. □