Mathematics 300

Quiz 11 **Name:** Answer Key

You must show your work to get full credit.

Theorem (Rational Root Test.). *Let a, b, c be integers with $a \neq 0$. Then if $r = p/q$ is a rational root of $ax^2 + bx + c = 0$ and r is in lowest terms, then $p \mid c$ and $q \mid a$.* $\square$

1. For the equation

$$x^2 + 4x - 2 = 0$$

(a) What are the possible rational roots?

Solution. Possible rational roots are: $\pm 1, \pm 2$ From the rational root test above we have that the rational roots are $r = p/q$ where $p \mod 2$ and $q \mid 1$. The only divisors of 2 are ± 1 and ± 1. Thus $p = \pm 1, \pm 2$. The only divisors of 1 are ± 1. Therefore the only possibilities for $r = p/q$ are $r = \pm 1$ and $r = \pm 2$. $\square$

(b) Prove that the equation $x^2 + 4x - 2 = 0$ has no rational roots.

Solution. Towards a contradiction assume that the equation has a rational root, call it r. By part (a) we have that this means that $r = 1$, $r = -1$, $r = 2$, or $r = -2$ and so one of these numbers would be a root of the equation. We now get a contradiction by showing that none of these are in fact roots:

$$\begin{aligned} (1)^2 + 4(1) - 2 &= 3 \neq 0 \\ (-1)^2 + 4(-1) - 2 &= -5 \neq 0 \\ (2)^2 + 4(2) - 2 &= 10 \neq 0 \\ (-2)^2 + 4(-2) - 2 &= -6 \neq 0 \end{aligned}$$

$\square$