Mathematics 300

Quiz 12 **Name:** <u>**Answer Key**</u>

You must show your work to get full credit.

1. Let $A = \{6x : x \in \mathbb{Z}\}$ and $B = \{k \in \mathbb{Z} : 12 \mid k\}$. Show $B \subseteq A$.

Solution. Let $b \in B$. Then $12 \mod b$ and thus there is some integer m such that $b = 12m$. But then $b = 6(2m) = 6x$ where $x = 2k$. Therefore $b \in A$. This shows that if $b \in B$, then $b \in A$ and and therefore that $B \subseteq A$. □

2. Let $A = \{2k + 5 : k \in \mathbb{Z}\}$ and $B = \{n \in \mathbb{Z} : n \text{ is odd}\}$. Show $A = B$.

Solution. We first show that $A \subseteq B$. Let $a \in A$. Then $a = 2k + 5$ for some $k \in \mathbb{Z}$. Thus

$$a = 2k + 5 = 2k + 4 + 1 = 2(k + 2) + 1 = 2n + 1$$

where $n = k + 2 \in \mathbb{Z}$. Therefore a is odd and thus $a \in B$. Thus $A \subseteq B$.

We now show $B \subseteq A$. Let $b \in B$. Then b is odd and therefore $b = 2n + 1$ for some $n \in \mathbb{Z}$. But then

$$b = 2n + 1 = 2n - 4 + 4 + 1 = 2(n - 2) + 5 = 2k + 5$$

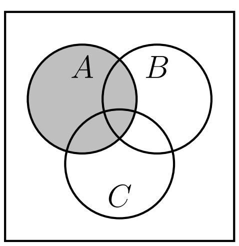
where $k = n - 2 \in \mathbb{Z}$. Therefore $b \in A$. Thus $B \subseteq A$.

But $A \subseteq B$ and $A \subseteq B$ together imply $A = B$ and we are done. □

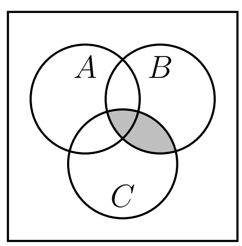
3. For any sets A and B show that $A \cap B \subseteq B$.

Proof. Let $x \in A \cap B$. Then by definition of intersection we have that $x \in A$ and $x \in B$. Thus $x \in B$. Therefore $x \in A \cap B$ implies $x \in B$ and thus $A \cap B \subseteq B$. □

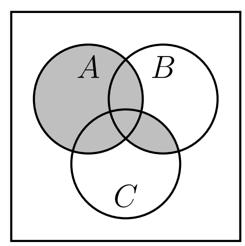
4. For any sets A, B, and C show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
Solution. For $A \cup (B \cap C)$ the Venn diagram can be found as follows



A

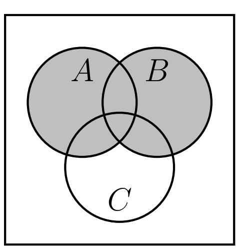


$B \cap C$

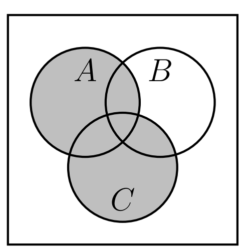


$A \cup (B \cap C)$

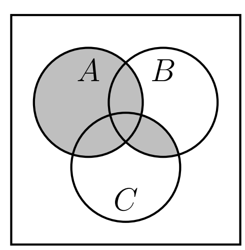
For $(A \cup B) \cap (A \cup C)$ we have



$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$

So the Venn diagrams are the same and thus the sets are equal.