Quiz 13   Name: Answer Key.

You must show your work to get full credit.

1. Prove that $\{6a + 4b : a, b \in \mathbb{Z}\} = \{2c : c \in \mathbb{Z}\}$.

Solution. So simplify notation let $A = \{6a + 4b : a, b \in \mathbb{Z}\}$ and $B = \{2c : c \in \mathbb{Z}\}$.

We first show $A \subseteq B$. Let $x \in A$. Then $x = 6a + 4b = 2(3a + 2b) = 2c$ where $c = 3a + 2b \in \mathbb{Z}$. Therefore $x \in B$ conpleting the proof that $A \subseteq B$.

Now we show $B \subseteq A$. Let $x \in B$. Then $x = 2c$ for some $c \in \mathbb{Z}$. Then $x = 2c = 6c + (-4)c = 6a + 4b$ where $a = c$ and $b = -c$ are integers. Therefore $x \in A$, which shows that $B \subseteq A$.

As we now have $A \subseteq B$ and $B \subseteq A$ we see that $A = B$.   □

2. Use Venn diagrams to show that for any sets A, B, and C that $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cap \overline{C}$.

Solution.

The Venn diagram for both is