Mathematics 300

Quiz 14 **Name: Answer Key**

You must show your work to get full credit.

1. Show that the two sets $\{9a - 6b : a, b \in \mathbb{Z}\}$ and $\{3c : c \in \mathbb{Z}\}$ are equal.

Solution. To simplify notation let $A = \{9a - 6b : a, b \in \mathbb{Z}\}$ and $B = \{3c : c \in \mathbb{Z}\}$.

We first show $A \subseteq B$. Let $x \in A$. Then $x = 9a - 6b$ for integers a and b. Then $x = 9a - 6b = 3(3a - 2b) = 3c$ where $c = 3a - 2b$ is an integer. Therefore $x \in B$ showing that $A \subseteq B$.

Now we show $B \subseteq A$. Let $x \in B$. Then $x = 3c$ for some integer c. Then $x = 3c = 9c - 6c = 9a - 6b$ where a and b are the integers $a = b = c$. Thus $x \in A$. This shows that $B \subseteq A$.

As $A \subseteq B$ and $B \subseteq A$ we conclude that $A = B$. □

2. Prove or give a disproof: every even number is the sum of two odd numbers.

Proof. This is true. To get a feel for what is happening let us look at some examples:

$$2 = 1 + 1, \quad 4 = 3 + 1, \quad ,6 = 5 + 1, \quad ,8 = 7 + 1, \quad 0 = -1 + 1, -2 = -3 + 1.$$

So let n be an even integer. Then $n = 2k$ for some integer k and we have

$$n = 2k = (2k - 1) + 1$$

and so n is the sum of the two odd numbers $2k - 1$ and 1. □