Mathematics 300

Quiz 16 **Name:** **Answer Key**

You must show your work to get full credit.

1. Use induction to show prove that $1 + 5 + 5^2 + \cdots + 5^n = \dfrac{5^{n+1} - 1}{4}$

Solution. We use as the base case $n = 0$ in which case the formula becomes

$$1 = \frac{5^{0+1} - 1}{4}$$

which reduces to the true statement that $1 = 1$. So the base case holds.

Now the induction step. Assume that the formula holds for n. That is

$$1 + 5 + 5^2 + \cdots + 5^n = \frac{5^{n+1} - 1}{4}. \tag{1}$$

We wish to show that this holds with n replaced by $n + 1$.

$$\begin{aligned}
1 + 5 + 5^2 + \cdots + 5^{n+1} &= (1 + 5 + 5^2 + \cdots + 5^n) + 5^{n+1} \\
&= \frac{5^{n+1} - 1}{4} + 5^{n+1} && \text{(by Equation (1))} \\
&= \frac{5^{n+1} - 1}{4} + \frac{4 \cdot 5^{n+1}}{4} \\
&= \frac{(1 + 4)5^{n+1} - 1}{4} \\
&= \frac{5^{(n+1)+1} - 1}{4}.
\end{aligned}$$

This shows that when the formula is true for n it is true for $n + 1$ and closes the induction. □