Mathematics 300

Quiz 17 **Name:** **Answer Key**

You must show your work to get full credit.

We have seen earlier in the term that if

$$a \equiv b \mod n \quad \text{and} \quad c \equiv d \mod n$$

then

$$ac \equiv bd \mod n.$$

1. Use the above to show

$$a \equiv b \mod$$

implies for all positive integers n

$$a^n \equiv b^n \mod n.$$

Solution. The base case is $n = 1$ where the statement is $a \equiv b \mod n$, which is what is given. So this holds.

For the induction step assume that

$$a^n \equiv b^n \mod n \tag{1}$$

holds. We will show this holds for with n replace by $n+1$. We are given that $a \equiv b \mod n$. Multiplying both sides of (1) by this we get

$$a \cdots a^n \equiv b \cdot b^n \mod n$$

that is

$$a^{n+1} \equiv b^{n+1} \mod n.$$

This closes the induction. □

2. Use Problem 1 to show that if that $7^n - 1$ is divisible by 6 for all positive integers n. *hint:* Showing that $7^n - 1$ is divisible by 6 is the same as showing $7^n - 1 \equiv 0 \mod 6$.

Solution. Note that $7 \equiv 1 \mod 6$ and therefore by Problem 1 we have $7^n \equiv 1^n \mod 6$. Thus

$$\begin{aligned} 7^n - 1 &\equiv 1^n - 1 \mod 6 \\ &\equiv 1 - 1 \mod 6 \\ &\equiv 0 \mod 6 \end{aligned}$$

But $7^n - 1 \equiv 0 \mod 6$ implies $6 \mid (7^n - 1)$ as required. □

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