Mathematics 300

Quiz 19 **Name: Answer Key**

You must show your work to get full credit.

1. Use

(1) $$\overline{B \cup C} = \overline{B} \cap \overline{C}$$

and induction to show

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n}$$

where $A_1, A_2, \ldots, A_n$ are subsets of some universal set.

Solution. We use as the base case $n = 2$ where the statement becomes

$$\overline{A_1 \cup A_n} = \overline{A}_n \cap \overline{A}_2$$

which is $\overline{B \cup C} = \overline{B} \cap \overline{C}$ with $B = A_1$ and $C = A_2$.

For the induction step assume that

(2) $$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A}_1 \cap \overline{A}_2 \cap \cdots \cap \overline{A}_n.$$

and use this to show that the result holds for with n repalaced by $n + 1$.

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n \cup A_{n+1}} = \overline{(A_1 \cup A_2 \cup \cdots \cup A_n) \cup A_{n+1}}$$

$$= \overline{(A_1 \cup A_2 \cup \cdots \cup A_n)} \cap \overline{A}_{n+1} \qquad \text{(Using (1) with } B = (A_1 \cdots \cup A_n) \text{ and } C = A_{n+1})$$

$$= \overline{A}_2 \cap \cdots \cap \overline{A}_n \cap \overline{A}_{n+1} \qquad \text{(Using Equation (2))}.$$

This completes the induction step. □

2. Concerning the Fibonacci sequence, prove that $F_1 + F_2 + F_3 + \cdots + F_n = F_{n+2} - 1$.

Solution. We use for the base case $n = 2$ when the equation becomes

$$F_1 + F_2 = F_{2+2} - 1.$$

As $F_1 = F_2 = 1$ and $F_4 = 3$ this becomes $1 + 1 = 3 - 1$ which is true. So the base case holds.

For the induction step assume

$$F_1 + F_2 + F_3 + \cdots + F_n = F_{n+2} - 1$$

holds. Add F_{n+1} to both sides of this to get:

$$F_1 + F_2 + F_3 + \cdots + F_n + F_{n+1} = F_{n+2} - 1 + F_{n+1} = (F_{n+1} + F_{n+2}) - 1 = F_{n+3} - 1 = F_{(n+1)+1} - 1$$

where we have used that $F_{n+1} + F_{n+2} = F_{n+3}$ which is the basic defining relation for the Fibonacci sequence. This completes the induction. □