Final   Name:_____

> ***You must show your work to get full credit.*** ***Problems are 8 points each***

1. Write out the set $\{x \in \mathbb{Z} : (x+2)(x-3) < 0\}$.   The set is _____

2. Let $A = \{0, 1\}$ and $B = \{1, 2, 3\}$. Write out the elements of the following:

$$A \cup B = \text{_____}$$

$$A \cap B = \text{_____}$$

$$B - A = \text{_____}$$

The power set of A is $\mathcal{P}(A) = \text{_____}$

3. Draw the Venn diagram for $(A \cap C) - B$.

4. (a) Make the truth tables for $P \implies Q$ and $\sim (P \wedge \sim Q)$.

   (b) Are $P \implies Q$ and $\sim (P \wedge \sim Q)$ logically equivalent? Your answer should involve at least one English sentence.

5. What is the negation of the statement "Every one taking this final will get an A on it."?

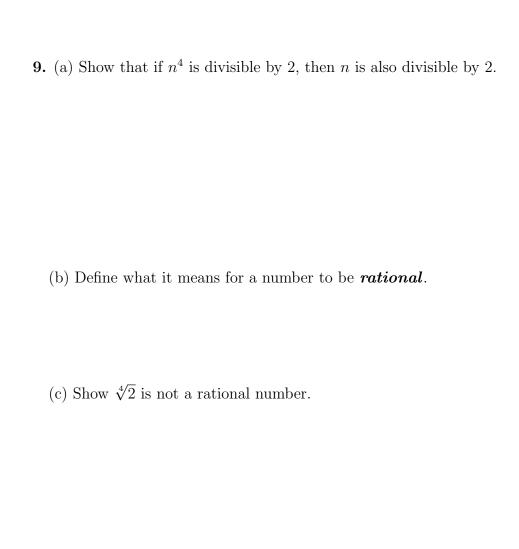**6.** Let for each of the integers $n = 1, 2, 3, \ldots$ let $A_n = \{1, 2, \ldots, n\}$. Then find:

$$\bigcup_{n=1}^{\infty} A_n = \underline{\hspace{3cm}}$$

$$\bigcap_{n=1}^{\infty} A_n = \underline{\hspace{3cm}}$$

7. (a) Define what it means for the integer x to be divisible by the integer m. (This definition should have some English and not just symbols.)

(b) Prove that if a and b are divisible by 3 then $4a^3 + 6ab$ is divisible by 27.

8. Give a proof or disproof that every multiple of 4 is the sum of two odd numbers.

9. (a) Show that if n^4 is divisible by 2, then n is also divisible by 2.

(b) Define what it means for a number to be **rational**.

(c) Show $\sqrt[4]{2}$ is not a rational number.

10. Show that α is irrational if and only if $3 + 2\alpha$ is irrational.

11. Let $A = \{15a + 10b : a, b \in \mathbb{Z}\}$ and $B = \{5c : c \in \mathbb{Z}\}$. Prove $A = B$.

12. Use that $10 \equiv 1 \mod 9$ to explain why
$$9{,}875{,}184 \equiv 9 + 8 + 7 + 5 + 1 + 8 + 4 \mod 9$$

13. Prove or give a disproof: There exist sets A and B such that $A = A \cap B$

14. Show that if $(x + 2y)^2 = x^2 + (2y)^2$, then $x = 0$ or $y = 0$.

15. (a) Define $x \equiv y \mod n$.

(b) Show that if $x \equiv y \mod n$, then for any integer a that $ax \equiv ay \mod n$.

16. (a) Explain briefly how and why a proof by induction works.

(b) Use induction to prove that for any real numbers a and r with $r \neq 1$ that
$$a + ar + ar^2 + \cdots + ar^n = \frac{a - ar^{n+1}}{1 - r}.$$

17. Use induction to show that $n^3 + 5n$ is divisible by 3 for all positive integers n.

18. On the set $A = \{1, 2, 3, 4\}$ we have an equivalence relation, R, what has the equivalence classes $\{1\}$, $\{2\}$, and $\{3, 4\}$. Write out R as a set of ordered pairs.

$$R = \underline{\hspace{3in}}$$

19. Given the addition and multiplication tables for the equivalence classes of integers mod 3.

Have a good holiday break.