Mathematics 172

Quiz 2 Name: Key

You must show your work to get full credit.

1. Show that for all integers x that $x^5 - 3x + 7$ is odd.

There are two cases.

case 1. x is even. Then $x \equiv 0 \mod 2$.

Whence $x^5 - 3x + 7 \equiv 0^5 - 3(0) + 7 \mod 2$

$\equiv 7 \mod 2$

$\equiv 1 \mod 2$

so x is odd.

case 2 x is odd. Then $x \equiv 1 \mod 2$.

Whence $x^5 - 3x + 7 \equiv 1^5 - 3 \cdot 1 + 7 \mod 2$

$\equiv 1 - 3 + 7 \mod 2$

$\equiv 5 \mod 2$

$\equiv 1 \mod 2$

so x is also odd in this case. done

2. Give a contrapositive proof that if $x^2 + 1$ is even, that x is odd.

The contrapositive is If x is even, then x^2+1 is odd.

Assume x is even. Then $x = 2k$ for some $k \in \mathbb{Z}$.

Then

$x^2 + 1 = (2k)^2 + 1$

$= 4k^2 + 1$

$= 2(2k^2) + 1$

and $2k^2 \in \mathbb{Z}$. Thus x^2+1 is odd. done