Mathematics 172

Quiz 3 Name: Alex

You must show your work to get full credit.

1. Give a contrapositive proof that if $x(x+5)$ is negative, then x is negative.
We will prove the contrapositive, that is
(a) State the contrapositive: If x is positive, then $x(x+5)$ is positive.

(b) So assume that x is positive and show that $x(x+5)$ is positive. *Hint:* If x is positive, then so is $x+5$ and the product of positive numbers is positive.

That is we wish to show $x>0$ implies $x(x+5)>0$.
So assume $x>0$. Then $x+5 > 0+5 > 0$.
Thus $x(x+5)>0$. done.

2. Show that if x^3 divisible by 3, then so is x.
We will prove the contrapositive, that is
(a) State the contrapositive:
If x is not divisible by 3 then x^3 is not divisible by 3
(or $3 \nmid x \Rightarrow 3 \nmid x^3$)

That is we need to show that if x is not divisible by 3, then x^3 is not divisible by 3. If $3 \nmid x$. Then there are two cases. $x \equiv 1 \mod 3$ and $x \equiv 2 \mod 3$.

Case 1: $x \equiv 1 \mod 3$. Then

$$\begin{aligned} x^3 &\equiv (1)^3 \mod 3 \\ &\equiv 1 \mod 3. \end{aligned}$$

Thus the remainder when x^3 is divided by 3 is 1 and thus x^3 is not divisible by 3.

Case 2: $x \equiv 2 \mod 3$. You fill in the details, which should involve using about as much English as I used above. Then

$$\begin{aligned} x^3 &\equiv 2^3 \mod 3 \\ &\equiv 8 \mod 3 \\ &\equiv 2 \mod 3 \end{aligned}$$

So the remainder when x^3 is divided by 3 is 2 and thus x^3 is not divisible by 3.