Mathematics 172

Quiz 4 Name: Key

You must show your work to get full credit.

1. Give a contrapositive proof that if x^3 is even, then x is even.

The contrapositive is: If x is odd, then x^3 is odd.

Assume x is odd. Then $x \equiv 1 \mod 2$.

Thus $x^3 \equiv 1^3 \mod 2$
$\equiv 1 \mod 2$.

So x^3 is odd.

2. Prove that $\sqrt[3]{2}$ is irrational. Towards a contradiction assume $\sqrt[3]{2}$ is rational, say

$$\sqrt[3]{2} = \frac{a}{b}$$

with $a, b \in \mathbb{Z}$ and the fraction $\frac{a}{b}$ in lowest terms.

Then $a = \sqrt[3]{2}\, b$

so $a^3 = 2b^3$

This shows a^3 is even and therefore by problem 1 a is even. Thus $a = 2k$ for some $k \in \mathbb{Z}$. Using this in $a^3 = 2b^3$ gives

$(2k)^3 = 2b^3$

$8k^3 = 2b^3$

$b^3 = 4k^3 = 2(2k^3)$

so b^3 is even and (again by problem 1) b is even. Thus $b = 2l$ for some $l \in \mathbb{Z}$.

But then $\frac{a}{b} = \frac{2k}{2l}$ is not in lowest terms, a contradiction.