Mathematics 172

Quiz 6 Name: Key

You must show your work to get full credit.

Proposition. $n^2 + 3$ is even if and only if n is odd.

1. Prove this. This means you have to prove two implications:

(a) If n is odd, then $n^2 + 3$ is even.

Assume n is odd, then $n \equiv 1 \mod 2$.

so $n^2+3 \equiv 1^2+3 \mod 2$

$\equiv 4 \mod 2$

$\equiv 0 \mod 2$

so n^2+3 is even.

(b) If $n^2 + 3$ is even, then n is odd.

We prove the contrapositive: If n is even, then n^2+3 is odd.

Assume n is even. Then $n \equiv 0 \mod 2$

so $n^2+3 \equiv 0^2+3 \mod 2$

$\equiv 3 \mod 2$

$\equiv 1 \mod 2$

Thus n^2+3 is odd.