Mathematics 300

Quiz 7 Name: Key

You must show your work to get full credit.

1. Let a, b, c, d integers and r a rational number with $cr + d \neq 0$. Show that $s = \dfrac{ar + b}{cr + d}$ is also a rational number. *Hint:* Start by assuming that $r = p/q$ for integers p and q and recall that to show s is rational you need to show that of the form $s = \dfrac{\text{integer}}{\text{integer}}$.

Assume $r = \frac{p}{q}$ with $p, q \in \mathbb{Z}$. then

$$s = \frac{ar+b}{cr+d} = \frac{a(\frac{p}{q})+b}{c(\frac{p}{q})+d} = \frac{a(\frac{p}{q})+b}{c(\frac{p}{q})+d} \cdot \left(\frac{q}{q}\right)$$

$$= \frac{ap+bq}{cp+dq}.$$

As $ap+bq, cp+dq \in \mathbb{Z}$, this shows s is a ratio of integers and thus rational

2. Let α be a real number. Show that α is irrational if and only if $3\alpha - 2$ is irrational. *Hint:* You are allowed to use the result of Problem 1.