Mathematics 300

Quiz 8 Name: Key

You must show your work to get full credit.

1. Show that if $(x+y)^2 = x^2 + y^2$, then $x = 0$ or $y = 0$.

Assue $(x+y)^2 = x^2+y^2$.

Then $\cancel{x^2} + 2xy + \cancel{y^2} = \cancel{x^2} + \cancel{y^2}$

$2xy = 0$

so $x = 0$ or $y = 0$.

2. For an integer a show that a is odd if and only if $a^2 \equiv 1 \mod 4$.

There are two implications.

1. If a is odd, then $a^2 \equiv 1 \mod 4$.

Assue a is odd. Then $a = 2k+1$ with $k \in \mathbb{Z}$.

$a^2 = (2k+1)^2 = 4k^2+4k+1 = 4(k^2+k)+1$

Thus $a^2 \equiv 4(k^2+k)+1 \equiv 1 \mod 4$.

2 If $a^2 \equiv 1 \mod 4$ then a is odd.

We prove the contrapositive: If a is even, then $a^2 \not\equiv 1 \mod 4$.

Assume a is even. Then $a = 2k$ for some $k \in \mathbb{Z}$.

$$\begin{aligned} a^2 &\equiv (2k)^2 \mod 4 \\ &\equiv 4k^2 \mod 4 \\ &\equiv 0 \mod 4 \\ &\not\equiv 1 \mod 4. \end{aligned}$$