Mathematics 300 Test 3

Name: _____

You are to use your own calculator, no sharing.

Show your work to get credit.   **All problems are 15 points.**

1. (a) Show that for any sets A and B that
$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B).$$
(It is ok to use Venn diagrams, but there should be at least one sentence of English saying why the diagrams are relevant.)

(b) Give an example to show that $A \cap B \neq A \cup B$. Here the answer should be two explicitly given sets, not just a Venn diagram.

2. (a) Prove or disprove: Every odd integer is the sum of an even integer and an odd integer. (Your answer should have at least one English sentence in it.)

(b) Prove or disprove. For integers a, b, and c if $a \mid bc$, then $a \mid b$ or $a \mid c$. (Your answer should have at least one English sentence in it.)

3. Prove that if $A = \{8a - 6b : a, b \in \mathbb{Z}\}$ and $B = \{2c : c \in \mathbb{Z}\}$ that $A = B$.

4. Recall that the Fibonacci numbers are defined by $F_1 = 1$, $F_2 = 1$ and
$$F_n = F_{n-1} + F_{n-2}$$
for all $n \geq 3$.
  (a) Compute the following: $F_3 = \underline{\hspace{1cm}}$ $F_4 = \underline{\hspace{1cm}}$ $F_5 = \underline{\hspace{1cm}}$ $F_6 = \underline{\hspace{1cm}}$
  (b) Use induction to prove
$$\sum_{k=1}^{n} F_k = F_{n+2} - 1.$$
Your proof should have enough English in it to explain what the various steps are.

5. Use induction to show that $n^3 + 5$ is divisible by 3 for all positive integers n. Your proof should have enough English in it to explain what the various steps are.

6. Let R be a relation on a set A. Define the following:

(a) R is **reflexive**.

(b) R is **symmetric**.

(c) R is **transtive**.

7. On the integers define a relation, R, by xRy if and only if $|x - y| = 1$.

(a) Is R reflexive _____. If not explain why.

(b) Is R symmetric _____. If not explain why.

(c) Is R translative _____. If not explain why.