Mathematics 300

Quiz 10 Name: Key

You must show your work to get full credit.

1. Define what it means for an integer to be ***even***.

An integer n is even if $n = 2a$ for some integer $a \in \mathbb{Z}$

2. Define what it means for an integer to be ***odd***.

An integer n is odd if $n = 2a+1$ for some integer $a \in \mathbb{Z}$

3. Prove that it x and y are odd integers, prove that $3x^2 - y^2$ is even.

Proof By definition of x and y being odd

$x = 2a+1$

$y = 2b+1$

for some integers $a, b \in \mathbb{Z}$. Then

$$\begin{aligned} 3x^2 - y^2 &= 3(2a+1)^2 - (2b+1)^2 \\ &= 3(4a^2+4a+1) - (4b^2+4b+1) \\ &= 12a^2+12a+3 - 4b^2-4b-1 \\ &= 12a^2+12a-4b^2-4b+2 \\ &= 2(6a^2+6a-2b^2-2b+1) \\ &= 2c \end{aligned}$$

where $c = 6a^2+6a-2b^2-2b+1 \in \mathbb{Z}$.

Thus $3x^2-y^2$ is even. done

4. Expand $(x-y)^3$.

$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$