Mathematics 300

Quiz 13 Name: Key

You must show your work to get full credit.

1. Define d is the ***greatest common divisor*** the integers a and b.

d is an integer that divides both a and b, and d is the largest integer that divides both a and b.

2. Define m is the ***least common multiple*** of the integers a and b.

m is an integer which is a multiple of both a and b, and is the smallest positive integer that is a multiple of both a and b.

3. State the ***division algorithm***.

Let a, and b be integers with $b > 0$. Then there are unique integers q and r such that

$a = qb + r$ and $0 \le r < b$.

4. Show for all integers n that $n^2 - 3n + 5$ is odd.

We consider two cases.

Case 1 n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$.

Then

$$
\begin{aligned}
n^2 - 3n + 5 &= (2k)^2 - 3(2k) + 5 \\
&= 4k^2 - 6k + 5 \\
&= 2(2k^2 - 3k + 2) + 1 \\
&= 2m + 1
\end{aligned}
$$

where $m = 2k^2 - 3k + 2 \in \mathbb{Z}$. Thus $n^2 - 3n + 5$ is odd

Case 2 n is odd. Then $n = 2k + 1$ for some integer $k \in \mathbb{Z}$.

$$
\begin{aligned}
n^2 - 3n + 5 &= (2k+1)^2 - 3(2k+1) + 5 \\
&= 4k^2 + 4k + 1 - 6k - 3 + 5 \\
&= 4k^2 - 2k + 3 \\
&= 2(2k^2 - k + 1) + 1 = 2m + 1
\end{aligned}
$$

where $m = 2k^2 - k + 1 \in \mathbb{Z}$. Thus $n^2 - 3n + 5$ is odd.

As every integer is either even or odd we have covered all cases.

done