Mathematics 300

Quiz 14

Name: Key

You must show your work to get full credit.

1. Give the contrapositive of the statement "If a is an integer and a^3 is even, then a is even".

If a is an integer and a is odd, then a^3 is odd.

2. Give a contrapositive proof that if a is an integer and a^3 is even, then a is even. (You may want to use the identity $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.)

Proof We will prove the contrapositive (as in problem 1).

Let a be odd, then $a = 2q+1$ for some integer $q \in \mathbb{Z}$. Then

$$\begin{aligned} a^3 &= (2q+1)^3 \\ &= (2q)^3 + 3(2q)^2 + 3(2q) + 1 \\ &= 8q^3 + 12q^2 + 6q + 1 \\ &= 2(4q^3 + 6q^2 + 3q) + 1 \\ &= 2k + 1 \end{aligned}$$

where $k = 4q^3 + 6q^2 + 3q$ is an integer. thus a^3 is odd.

done