Mathematics 300

Quiz 15 Name: Key

You must show your work to get full credit.

1. Let n be a positive integer and $a.b \in \mathbb{Z}$. Define what it means for a to be ***congruent to*** b modulo n (in symbols $a \equiv b \pmod{n}$).

$a \equiv b \pmod{n}$ if $n \mid (b-a)$

2. State the division algorithm and in particular what are the ***quotient*** and ***remainder*** if b is divide into a. If a and b are integers and $b > 0$, then there are unique integers q and r such that

$a = qb + r \quad 0 \le r < b$

q = the <u>quotient</u> of a divided by b

r = the <u>remainder</u> when a is divided by b

3. Show that if a and b both have remainder 3 when divided by 7, then $a \equiv b \pmod{7}$.

If a is divided by 7 then

$a = 7q + r$ where $0 \le r < r$

and r is the remainder.

When b is divided by 7 we have

$b = 7q_1 + r_1$

where r_1 is the remainder. The hypothesis is that $r_1 = r$. So

$$
\begin{aligned}
b - a &= 7q + r - (7q_1 + r_1) \\
&= 7(q - q_1) + (r - r_1) \\
&= 7(q - q_1) + 0 \qquad (\text{as } r = r_1) \\
&= 7(q - q_1) \\
&= 7k
\end{aligned}
$$

where $k = q - q_1 \in \mathbb{Z}$. Thus $7 \mid (b-a)$. Therefore $a \equiv b \bmod 7$.

<u>done</u>