Mathematics 300

**Quiz 16** Name: Key

***You must show your work to get full credit.***

**1.** Let $n$ be a positive integer and $a, b \in \mathbb{Z}$. Define $a \equiv b \pmod{n}$.

$a \equiv b \bmod n$ means $n \mid (a-b)$

or

$a \equiv b \bmod n$ means there is an integer $q$ such that $a - b = qn$.

**2.** If $a$ and $b$ have the opposite parity, then $b \equiv a + 1 \pmod{2}$.

Case 1 $a$ is even and $b$ is odd. Then there are integers $k$ and $l$ such that

$a = 2k,\ b = 2l + 1$

Then $b - (a+1) = 2l + 1 - (2k+1)$

$= 2(l-k) = 2q$

and $q = l - k \in \mathbb{Z}$. Thus $2 \mid (b-(a+1))$ and so $b \equiv a+1 \bmod 2$

Case 2 $a$ is odd and $b$ is even. Then there are integers $k$ and $l$ such that

$a = 2k+1$ and $b = 2l$.

whence

$b - (a+1) = 2l - (2k+1+1)$

$= 2l - 2k - 2 = 2(l - k - 1) = 2q$

where $q = l - k - 1 \in \mathbb{Z}$.

Thus $2 \mid (b-(a+1))$ and so $b \equiv a+1 \bmod 2$

**3.** Use the identity $x^4 - y^4 = (x-y)(x^3 + x^2y + xy^2 + y^3)$ to prove that if $x \equiv y \pmod{n}$, then $x^4 \equiv y^4 \pmod{n}$. If $x \equiv y \bmod n$, then there is an integer $q$ such that $x - y = qn$.

Thus

$x^4 - y^4 = (x-y)(x^3 + x^2y + xy^2 + y^3)$

$= qn(x^3 + x^2y + xy^2 + y^3)$

$= kn$

where $k = q(x^3 + x^2y + xy^2 + y^3) \in \mathbb{Z}$. So

$x^4 \equiv y^4 \bmod n$.