Mathematics 300

Quiz 17 Name: Key

You must show your work to get full credit.

1. Define r is a ***rational number***.

r is a rational number if $r = \frac{a}{b}$ where a and b are integers and $b \neq 0$.

2. Prove that if r is a rational number then so is $s = \dfrac{r+2}{r^2+3}$.

Proof We are given r is a rational number, so

$$r = \frac{a}{b}$$

for some $a, b \in \mathbb{Z}$ and with $b \neq 0$. Using this in the formula for s gives

$$\begin{aligned} s &= \frac{r+2}{r^2+3} \\ &= \frac{\left(\frac{a}{b}\right)+2}{\left(\frac{a}{b}\right)^2+3} \\ &= \frac{\left(\frac{a}{b}+2\right)\cdot b^2}{\left(\left(\frac{a}{b}\right)^2+3\right)\cdot b^2} \\ &= \frac{ab+2b^2}{a^2+3b^2} \\ &= \frac{p}{q} \end{aligned}$$

where $p, q \in \mathbb{Z}$ and $q \neq 0$. ($q \neq 0$ because $b \neq 0$ and $q = a^2+3b^2 \geq 3b^2 > 0$). So $s = \frac{p}{q}$ is rational. Done