Mathematics 300

Quiz 21 **Name:**________________________________

You must show your work to get full credit.

Let a be an integer. Prove that $a^3 + 2a + 1$ is odd if and only if a is even.

Proof. We need to prove each of the two implications:

(A) If $a^3 + 2a + 1$ is odd, then a is even.

(B) If a is even, then $a^3 + 2a + 1$ is odd.

Proof of (A). We prove the contrapositive: If a is odd, then $a^3 + 2a + 1$ is even. Assume that a is odd. Then

$$a \equiv 1 \pmod 2$$

and therefore

$$\begin{aligned} a^3 + 2a + 1 &\equiv 1^3 + 2 \cdot 1 + 1 &\pmod 2 \\ &\equiv 4 &\pmod 2 \\ &\equiv 0 &\pmod 2 \end{aligned}$$

Thus $a^3 + 2a + 1$ is even as required.

Proof of (B). Assume that a is even. Then

$$a \equiv 0 \pmod 2$$

Therefore

$$\begin{aligned} a^3 + 2a + 1 &\equiv 0^2 + 2 \cdot 0 + 1 &\pmod 2 \\ &\equiv 1 &\pmod 3 \end{aligned}$$

and therefore $a^3 + 2a + 1$ is odd. □