Mathematics 300

Quiz 22 Name:____________________

You must show your work to get full credit.

Proposition. *If a is an integer and $3a \equiv 2 \pmod 5$, then $a \equiv 4 \pmod 5$.*

Proof. This is based on noting that $2 \cdot 3 = 6 \equiv 1 \pmod 5$. So assume that

$$3x \equiv 2 \pmod 5.$$

Multiply both sides of this concurrence by 2 to get

$$2 \cdot 3a \equiv 2 \cdot 2 \pmod 5$$

This simplifies to

$$a \equiv 4 \pmod 5$$

Which is exactly what we wanted to show. $\square$

1. Use the idea above to what if x is an integer and $3x \equiv 4 \pmod 7$, then $x \equiv 6 \pmod 7$. *Hint:* $3 \cdot 5 = 15 \equiv 1 \pmod 7$.

Proof Assume that

$3x \equiv 4 \mod 7$

multiple both sides by 5 to get

$5 \cdot 3x \equiv 5 \cdot 4 \mod 7$

i.e. $15x \equiv 20 \mod 7$

But $15 \equiv 1 \mod 7$, $24 \equiv 6 \mod 7$

So $15x \equiv 20 \mod 7$ becomes

$x \equiv 0 \mod 7$. done