Mathematics 300

Quiz 25 Name: Key

You must show your work to get full credit.

Recall that the ***absolute value*** of a real number x is defined to be

$$|x| = \begin{cases} x, & x \geq 0; \\ -x, & x < 0. \end{cases}$$

1. Prove that for all real numbers x that $|-3x| = 3|x|$.

Because $|x|$ is defined by cases, it makes sense to use cases to prove this.

Case 1 $x=0$. Then $|x|=0$, so $3|x|=3(0)=0$. Also $-3x=-3(0)=0$ and so

$$|-3x| = |0| = 0 = 3(0) = 3|x|.$$

Case 2 $x>0$. Then $|x|=x$ and so $3|x|=3x$. Then $-3x<0$ so $|-3x| = -(-3x) = 3x$. So in this case

$$|-3x| = 3x = 3|x|$$

Case 3 $x<0$. Then $-3x>0$ so $|-3x|=-3x$. Also $x<0$ so $|x|=-x$. Thus

$$|-3x| = -3x = 3(-x) = 3|x|.$$

Thus in all cases $|-3x| = 3|x|$.

2. Write an English sentence or two explaining why 40 is the sum of two prime numbers.

The numbers 3 and 37 are both prime and their sum is $3+37=40$. So 40 is the sum of two prime numbers.

Remark: 1 is not a prime number.

3. Use that $10 \equiv 1 \pmod 9$ to explain why

$$3{,}427 \equiv 3+4+2+7 \pmod 9.$$

Recall that $3{,}427 = 3(10)^3+4(10)^2+2(10)+7$. Therefore using properties on congruences

$$\begin{aligned} 3{,}427 &= 3(10)^3+4(10)^2+2(10)+7 \\ &\equiv 3(1)^3+4(1)^2+2(1)+7 \pmod 9 \\ &\equiv 3+4+2+7 \pmod 9 \end{aligned}$$