Mathematics 300

Quiz 26 Name: Key

You must show your work to get full credit.

There is a rational root test for polynomials of degree higher than two. Here it is for polynomials of degree three.

Theorem (Rational root test). *Let a_0, a_1, a_2, and a_3 be integers with $a_3 \neq 0$. Let $p = \frac{p}{q}$ be a rational root of*

$$a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

with $\frac{p}{q}$ in lowest terms. (That is $\gcd(a, b) = 1$) Then

$$p \mid a_0 \quad \textit{and} \quad q \mid a_3$$

□

1. What are the possible rational roots of $x^3 - 7 = 0$?

Here $a_0 = 1$ and $a_3 = -7$ so a rational root $r = \frac{p}{q}$ will have
$p \mid 1$ (so $p = \pm 1$)
and $q \mid 7$ (so $q = \pm 1$ or $q = \pm 7$)

Thus $r = \frac{p}{q}$ is $r = \pm 1$ or $r = \pm 7$

Possible rational roots are: ± 1, ± 7

2. Explain why $x^3 - 7 = 0$ has no rational roots.

The only possible rational roots are $r = 1, -1, 7, -7$ we now try all of these

$1^3 - 7 = -6 \neq 0$

$(-1)^3 - 7 = -8 \neq 0$

$7^3 - 7 = 7(7^2 - 1) = 7(48) \neq 0$

$(-7)^3 - 7 = -7((-7)^2 - 1) = -7(48) \neq 0$.

Since none of these are roots, all roots must be irrational.

3. Prove $\sqrt[3]{7}$ is irrational.

If $x = \sqrt[3]{7}$, then

$x^3 = 7$

and so $x^3 - 7 = 0$.

Thus $x = \sqrt[3]{7}$ is a root of $x^3 - 7 = 0$

and since $x^3 - 7 = 0$ has no rational roots, it must be irrational.