Mathematics 300

Quiz 27 Name: Key.

You must show your work to get full credit.

For the following give a proof or a disproof.

1. The reciprocal of any irrational number is irrational.

True. Towards a contradiction assume there is an irrational number α such that $\frac{1}{\alpha}$ is rational. Then

$$\frac{1}{\alpha} = \frac{p}{q} \qquad p, q \in \mathbb{Z} \text{ and } q \neq 0.$$

Then $\alpha = \frac{q}{p}$. Thus α is rational, contradicting the assumption that it is irrational.

2. The square of any irrational number is irrational.

False. Let $\alpha = \sqrt{2}$. Then α is irrational. But $\alpha^2 = 2$ is rational.

3. For any real numbers $|x + y| = |x| + |y|$.

False Let $x = 1$, $y = -1$, then

$$|x+y| = |1+(-1)| = 0 \neq 1+1 = |x|+|y|$$

4. The number 33 is the sum of two primes.

True The numbers 2 and 29 are prime and their sum is $2+29=31$.

5. The number 35 is the sum of two primes.

False: Assume towards a contradiction that $35=p+q$ with both p and q prime. As the sum of two odd numbers is even and 35 is odd, at least one of p or q must be even. Say that p is even. Then $p=2$ because 2 is the only even prime. Then $35=p+q=2+q$ so $q=35-2=33=3\cdot 11$ is not prime a contradiction (The case q is even works just the same)

6. Let $A=\{x\in\mathbb{Z}:4\mid x\}$ and $B=\{n^2:n\in\mathbb{Z}$ and n is even$\}$. Then $B\subseteq A$.

True Let $b\in B$. Then $b=n^2$ where n is even. As n is even $n=2k$ for some $k\in\mathbb{Z}$. Thus

$$b=n^2=(2k)^2=4k^2 \text{ and } k^2\in\mathbb{Z}.$$

Thus $4\mid b$. Therefore $b\in A$.

This shows $B\subseteq A$.

7. The sum of two irrational numbers is irrational.

False Let $\alpha=\sqrt{2}$, $\beta=-\sqrt{2}$. Then both α and β are irrational. But their sum is

$$\alpha+\beta=\sqrt{2}+(-\sqrt{2})=0$$

and 0 is a rational number.