Mathematics 300

Quiz 29 Name: Key

You must show your work to get full credit.

Use induction to show that for all real numbers $x \neq -1$ integers $n \geq 1$ that

$$1 + x + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

Be sure to state the base case and the induction hypothesis.

The base case is $n=1$, in which case the identity becomes $1+x = \frac{1-x^{1+1}}{1-x}$

But $\frac{1-x^{1+1}}{1-x} = \frac{1-x^2}{1-x} = \frac{(1-x)(1+x)}{1-x} = 1+x$

which is true.

The induction step is that

S_n: $1+x+\cdots+x^n = \frac{1-x^{n+1}}{1-x}$ holds.

Add x^{n+1} to both sides of this to get

$$1+x+\cdots+x^n+x^{n+1} = \frac{1-x^{n+1}}{1-x} + x^{n+1}$$

$$= \frac{1-x^{n+1}}{1-x} + \frac{(1-x)\cdot x^{n+1}}{1-x}$$

$$= \frac{1-x^{n+1} + x^{n+1} - x^{n+2}}{1-x}$$

$$= \frac{1-x^{n+2}}{1-x}$$

$$= \frac{1-x^{n+1+1}}{1-x}$$

so S_{n+1} holds. This closes the induction and completes the proof.