Mathematics 300

Quiz 30 Name: Key

You must show your work to get full credit.

1. Show that for any $n \geq 8$ it is possible to put exactly n¢ on a letter using only 3¢ and 4¢ stamps.

Base case $n = 8$. Then $8 = 4+4$ so we can use two 4¢ stamps.

Induction hypothesis we can put k ¢ on a letter with $k \geq 8$.

If the k ¢ letter has a 3 ¢ stamp take it off and add a 4 ¢ stamp to get $k-3+4 = (k+1)$ ¢.

If the k ¢ letter has no 3 ¢ stamps, all the stamps are 4¢ stamps. As $k \geq 8$ there are at least 2 of them. So take out 2 4¢ stamps and add on 3 3 ¢ stamps to get $k-2(4)+3(3) = k+1$ ¢. This finishes the induction.

2. If $g(n) = 2g(n-1)$ and $g(1) = 6$, prove that $g(n) = 3(2)^n$ for all integers $n \geq 1$.

Base case: $n=1$ $3(2)^1 = 6 = g(1)$. So this holds.

Induction hypothesis S_k: That $g(k) = 3(2)^k$. Then

$$g(k+1) = 2g(k) = 2\,(3(2)^k) = 3(2)^{k+1}.$$

This finishes the induction.